Complex and Backward-Wave Modes in Inhomogeneously and Anisotropically Filled Waveguides

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Abstract —A rigorous analysis of lossless inhomogeneously and anisotropically filled waveguides of arbitrarily shaped cross section is presented. The mode propagation constants squared appear as eigenvalues of a real infinite-dimensional characteristic matrix, which is, in the general case, nonsymmetric. Complex conjugate pairs of eigenvalues are then possible, which give rise to complex modes. Modes at cutoff are shown to be either TE or TM with real cutoff frequencies. An investigation of the power flow shows that backward-wave modes may exist as well. Different orthogonality relations are derived from which the power coupling between complex modes is investigated.

I. Introduction

A BACKWARD WAVE is one in which the energy flows in the opposite direction to the wavefronts. Since, in the absence of reflections, energy must travel away from the generator, the wavefronts of a backward wave travel toward the generator. These waves have been known for a long time to propagate in periodic structures (see, e.g., [1]). The possibility of backward-wave modes in a circular waveguide coaxially loaded by a dielectric rod was first reported in [2]. The conditions under which backward-wave modes with unity azimuthal dependence can exist in these waveguides have been investigated (e.g., [3], [4]). It has been shown there that the degeneracy of the TE_{1m} and TM_{1m} modes at cutoff is associated with backward-wave propagation above cutoff. Potential applications of backward-wave modes in such waveguides have been discussed in [5]. In [6] and [7], it has been shown that backward-wave modes can also exist in semicircular, rectangular, and trough waveguides with dielectric inserts. Experimental verifications of backward-wave propagation in some of these structures have been reported (e.g. [7], [8]).

Complex modes are modes with complex propagation constants which can be supported by lossless guiding structures. Due to the lossless nature of the structure supporting such modes, they always exist in pairs, with complex conjugate propagation constants of opposite sign. The electric field of one mode of any pair does not couple to the magnetic field of the same mode. Instead, it couples to the magnetic field of the other mode in such a way that the total power carried by the two modes is purely reactive. Complex modes in a circular waveguide containing a coaxial dielectric rod were first predicted in [9]. It was shown there that the appearance of a backward-wave mode in a certain frequency band is associated with the appearance of complex modes in a lower frequency band. It has also been shown that complex modes can occur under certain conditions even if there is no frequency range in which backward-wave modes can propagate. More theoretical and experimental investigations on complex modes in dielectric loaded circular waveguides have been reported (e.g., [10]–[13]).

Complex modes in a shielded rectangular dielectric image guide, which can be considered as a rectangular waveguide with rectangular dielectric insert, have been reported in [14]. The relevance of complex modes, in conjunction with the analysis of the transition from a rectangular waveguide to a shielded dielectric image guide, has been described in [15], which the reader can refer to for a deeper understanding of the interesting characteristics of complex modes.

We have recently shown that both complex and backward-wave modes can exist in finlines [16], [17]. The effect of ignoring complex modes on the completeness of the set of normal modes has been investigated in [18] in conjunction with finline discontinuity problems. It has been stated in [17], without proof, that complex and backward-wave modes are believed to exist in any planar guiding structure with closed conducting boundaries.

In this contribution, this statement will be proved rigorously for the general case of inhomogeneously and anisotropically filled waveguides with arbitrarily shaped cross sections.

II. Inhomogeneously Filled Waveguides

Consider an inhomogeneously filled waveguide of arbitrarily shaped cross section S and perfectly conducting boundaries, as shown in Fig. 1. The longitudinal and the transverse coordinate vectors are $z\hat{\boldsymbol{k}}$ and $\boldsymbol{r}$, respectively. The filling medium is assumed to be lossless and to have a real transversally dependent relative permittivity and a constant relative permeability $\epsilon_r = \epsilon_r(\boldsymbol{r})$ and $\mu_r = 1$, respectively. The transverse inhomogeneity of ϵ_r can be treated as a polarization current $\boldsymbol{J}$ exciting the corresponding empty waveguide. This current is then given by

$$\boldsymbol{J} = j\omega\epsilon_0(\epsilon_r(\boldsymbol{r}) - 1)\boldsymbol{E}. \qquad (1)$$

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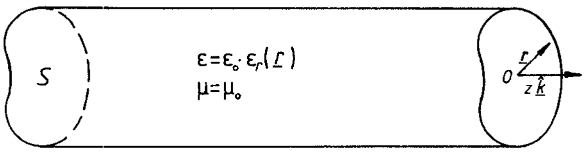

Fig. 1. An inhomogeneously filled waveguide with an arbitrarily shaped cross section.

Let the axial electric (magnetic) field of the nth TM (TE) mode of the empty waveguide be $e_{zn}(\boldsymbol{r})\cdot\exp(-\gamma_{ne}z)$ ($h_{zn}(\boldsymbol{r})\cdot\exp(-\gamma_{nh}z)$), with corresponding cutoff wavenumber $k_{ne}(k_{nh})$. Then

$$\nabla_t^2 e_{zn} + k_{ne}^2 e_{zn} = 0 \quad \gamma_{ne}^2 = k_{ne}^2 - k_0^2$$
$$\nabla_t^2 h_{zn} + k_{nh}^2 h_{zn} = 0 \quad \gamma_{nh}^2 = k_{nh}^2 - k_0^2 \tag{2}$$

where subscripts t and z refer to transverse and longitudinal components, respectively, and $k_0^2 = \omega^2\mu_0\epsilon_0$. Due to the orthogonal properties of these modes [19], they may be normalized according to

$$\int_S e_{zn}e_{zm}\,dS = \delta_{nm} \quad \int_S h_{zn}h_{zm}\,dS = \delta_{nm} \tag{3}$$

where δ_{nm} is the Kronecker delta. It will then be possible to choose e_{zn} and h_{zn} as real functions.

Expanding the transverse and longitudinal components of the electromagnetic field with respect to the TM and TE normal modes of the corresponding empty waveguide, substituting these expansions into Maxwell's equations, and making use of the orthogonality relations (3), one obtains the interrelations between the different expansion coefficients as well as their relations to the exciting current $\boldsymbol{J}$. This procedure is well described in [20]; thus, only final expressions are given here.

The transverse and longitudinal components of the electromagnetic field are expanded as

$$\boldsymbol{E}_t = e^{-j\beta z}\Big\{\sum_n(-A_n/k_{ne})\nabla_t e_{zn} + \sum_n(j\omega\mu_0 B_n/k_{nh})(\hat{\boldsymbol{k}}\times\nabla_t h_{zn})\Big\}$$
$$\boldsymbol{H}_t = e^{-j\beta z}\Big\{\sum_n(-j\beta B_n/k_{nh})\nabla_t h_{zn} + \sum_n(j\omega\epsilon_0 D_n/k_{ne})(\nabla_t e_{zn}\times\hat{\boldsymbol{k}})\Big\}$$
$$E_z = e^{-j\beta z}\sum_n\big\{(j\beta A_n + k_0^2 D_n)/k_{ne}\big\}e_{zn}$$
$$H_z = e^{-j\beta z}\sum_n k_{nh}B_n h_{zn} \tag{4}$$

where a z dependence $e^{-j\beta z}$ is assumed; $\hat{\boldsymbol{k}}$ is the unit vector in the longitudinal direction and A_n, B_n, and D_n are series expansion coefficients. The expansion coefficients are related to the exciting current $\boldsymbol{J}$ by

$$e^{-j\beta z}(\gamma_{ne}^2 D_n - j\beta A_n) = (k_{ne}/j\omega\epsilon_0)\int_S J_z e_z\,dS$$
$$e^{-j\beta z}(A_n - j\beta D_n) = (1/j\omega\epsilon_0 k_{ne})\int_S \boldsymbol{J}_t\cdot\nabla_t e_{zn}\,dS$$

and

$$e^{-j\beta z}(\beta^2 + \gamma_{nh}^2)B_n = (-1/k_{nh})\int_S \boldsymbol{J}_t\cdot(\hat{\boldsymbol{k}}\times\nabla_t h_{zn})\,dS. \tag{5}$$

Using (1)–(5), it can be shown that the expansion coefficients A_n, B_n, and D_n are related by the following infinite system of linear equations:

$$([I] - k_0^2[S])\boldsymbol{D} = j\beta[S]\boldsymbol{A} \tag{6a}$$
$$[R^e]\boldsymbol{A} - j\beta\boldsymbol{D} = -j\omega\mu_0[T]\boldsymbol{B} \tag{6b}$$
$$(k_0^2[R^h] - [\Lambda^h])\boldsymbol{B} - \beta^2\boldsymbol{B} = j\omega\epsilon_0[T]^t\boldsymbol{A}. \tag{6c}$$

$\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$ are column vectors with elements A_n, B_n, and D_n, respectively, $[\Lambda^h]$ is a diagonal matrix with elements k_{nh}^2, $[R^e]$, $[R^h]$, and $[S]$ are real symmetric matrices, $[T]^t$ is the transposed matrix of $[T]$, both being real, and $[I]$ is the identity matrix. All matrices and column vectors of (6) have infinite dimensions. The elements of $[R^e]$, $[R^h]$, $[S]$, and $[T]$ are given by

$$R_{nm}^e = (1/k_{ne}k_{me})\int_S \epsilon_r(\nabla_t e_{zn}\cdot\nabla_t e_{zm})\,dS$$
$$R_{nm}^h = (1/k_{nh}k_{mh})\int_S \epsilon_r(\nabla_t h_{zn}\cdot\nabla_t h_{zm})\,dS$$
$$S_{nm} = (1/k_{ne}k_{me})\int_S \epsilon_r(e_{zn}e_{zm})\,dS$$
$$T_{nm} = (1/k_{ne}k_{mh})\int_S \epsilon_r(\nabla_t e_{zn}\times\nabla_t h_{zm})\cdot d\boldsymbol{S}. \tag{7}$$

At this stage, it is important to state that the above-mentioned formulation does not at all represent an easier alternative for the analysis of the well-known special cases of inhomogeneously filled waveguides, e.g., dielectric-slab-loaded rectangular waveguide or dielectric-rod-loaded circular waveguide. Other methods, e.g., [21]–[23], are much more promising as far as the special cases for which they are formulated are concerned. The present formulation, on the other hand, is valid for the general case of waveguides with arbitrarily shaped cross section and arbitrary inhomogeneity of the filling medium. Furthermore, the general mode characteristics such as completeness and orthogonality can be studied directly by using the present approach rather than the other computationally oriented ones.

A. The Homogeneously Filled Waveguide as a Special Case

If ϵ_r is constant over the waveguide cross section, the matrices appearing in (6) take simple forms, namely $[R^e] = \epsilon_r[I]$, $[R^h] = \epsilon_r[I]$, $[S] = \epsilon_r[\Lambda^e]^{-1}$, and $[T] = 0$, where $[\Lambda^e]$ is a diagonal matrix with elements k_{ne}^2. Then (6) reads

$$(\epsilon_r k_0^2[I] - [\Lambda^e])\boldsymbol{D} = \beta^2\boldsymbol{D} \tag{8a}$$
$$(\epsilon_r k_0^2[I] - [\Lambda^h])\boldsymbol{B} = \beta^2\boldsymbol{B}. \tag{8b}$$

A solution of these equations is either $\boldsymbol{A} = 0 = \boldsymbol{D}$, $B_n = B_m\delta_{nm}$, and $\beta^2 = \epsilon_r k_0^2 - k_{mh}^2$, which represents the mth TE mode, or $\boldsymbol{B} = 0$, $D_n = D_m\delta_{nm}$, $A_n = (j\beta/\epsilon_r)D_m\delta_{nm}$, and $\beta^2 = \epsilon_r k_0^2 - k_{me}^2$, which represents the mth TM mode.

B. Modes at Cutoff

Setting $\beta = 0$, (6) reads

$$\{[\Lambda^h] - k_0^2([R^h] - [T]^t[R^e]^{-1}[T])\}\boldsymbol{B} = 0 \quad (9a)$$

$$([I] - k_0^2[S])\boldsymbol{D} = 0. \quad (9b)$$

These equations mean that there exist two sets of modes at cutoff. The first set has $\boldsymbol{D} = 0$ and $\boldsymbol{A} \neq 0, \boldsymbol{B} \neq 0$; the second set has $\boldsymbol{A} = 0$, $\boldsymbol{B} = 0$, and $\boldsymbol{D} \neq 0$. Substituting $\beta = 0$ in (4), we find that $\boldsymbol{H}_t = 0$, $E_z = 0$ for the modes of the first set, whereas $\boldsymbol{E}_t = 0$, $H_z = 0$ for the modes of the second set. We can then conclude that modes at cutoff are either TE characterized by (9a) or TM characterized by (9b). Due to the symmetry of the real matrices $[\Lambda^h]$, $[R^h]$, $([T]^t[R^e]^{-1}[T])$, and $[S]$, the squares of the cutoff wavenumbers of both TE and TM modes, which appear as the eigenvalues of (9a) and (9b), respectively, are all real (see, e.g., [24]).

C. Complex Modes

Eliminating $\boldsymbol{D}$ by using (6a), we get the following eigenvalue equation:

$$\begin{bmatrix} (k_0^2[I] - [S]^{-1})[R^e] & j\omega\mu_0(k_0^2[I] - [S]^{-1})[T] \\ -j\omega\epsilon_0[T]^t & (k_0^2[R^h] - [\Lambda^h]) \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{B} \end{bmatrix} = \beta^2 \begin{bmatrix} \boldsymbol{A} \\ \boldsymbol{B} \end{bmatrix}. \quad (10)$$

To have a real characteristic matrix, we use the transformation

$$\boldsymbol{A} = \sqrt{\lambda}\boldsymbol{A}' \quad \boldsymbol{B} = (j/\sqrt{\lambda})\boldsymbol{B}' \quad (11)$$

where λ is a real positive constant. The eigenvalue equation (10) then reads

$$\begin{bmatrix} (k_0^2[I] - [S]^{-1})[R^e] & (-\omega\mu_0/\lambda)(k_0^2[I] - [S]^{-1})[T] \\ -\omega\epsilon_0\lambda[T]^t & (k_0^2[R^h] - [\Lambda^h]) \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{A}' \\ \boldsymbol{B}' \end{bmatrix} = \beta^2 \begin{bmatrix} \boldsymbol{A}' \\ \boldsymbol{B}' \end{bmatrix}. \quad (12)$$

The two diagonal submatrices of the characteristic matrix in (12) are real and symmetric. For the total characteristic matrix to be symmetric, both matrices $[S]$ and $[T]$ must satisfy

$$([S]^{-1} - (k_0^2 - \lambda^2 Y_0^2)[I])[T] = 0 \quad (13)$$

where Y_0 is the free-space intrinsic admittance. Equation (13) means that either $[T] = 0$ or the different columns of $[T]$ are the eigenvectors of $[S]^{-1}$ which correspond to the same eigenvalue $(k_0^2 - \lambda^2 Y_0^2)$ [24]. The vanishing of $[T]$ leads to the vanishing of the off-diagonal submatrices of the characteristic matrix in (12). The TM and TE parts of the field, which are represented by $\boldsymbol{A}'$ and $\boldsymbol{B}'$, respectively, must then be decoupled. Modes of TE or TM type exist, however, only in a very limited number of special cases, e.g., in homogeneously filled waveguides or as modes without azimuthal dependence in rotational symmetrically filled circular waveguides (as will be shown later). The alternative condition on the columns of $[T]$ cannot be fulfilled even for such simple structures as the dielectric-slab-loaded rectangular waveguide. We can conclude then that the characteristic matrix of (12) is, in general, real and nonsymmetric. This means that its eigenvalues (β^2) are either real or complex conjugate pairs, because they are zeros of a polynomial of infinite order which has real coefficients. Complex conjugate pairs of β^2 characterize the so-called complex modes.

D. Orthogonality Relations

Coupling between modes of lossless guiding structures can be described by either of the two quantities

$$P_{ij} = \int_S (\boldsymbol{e}_i \times \boldsymbol{h}_j^*) \cdot \boldsymbol{dS} \quad Q_{ij} = \int_S (\boldsymbol{e}_i \times \boldsymbol{h}_j) \cdot \boldsymbol{dS} \quad (14)$$

where $\boldsymbol{e}_i$ and $\boldsymbol{h}_j$ are the transverse electric and magnetic field vectors of the ith and jth modes, respectively, and * denotes the complex conjugate. Using (2)–(4) and (6a), it is readily proved that

$$P_{ij} = \beta_j^* k_0 (Z_0 \boldsymbol{B}_i^t \boldsymbol{B}_j^* - Y_0 \boldsymbol{A}_i^t [C] \boldsymbol{A}_j^*)$$

$$Q_{ij} = -\beta_j k_0 (Z_0 \boldsymbol{B}_i^t \boldsymbol{B}_j + Y_0 \boldsymbol{A}_i^t [C] \boldsymbol{A}_j) \quad (15)$$

where $[C] = ([S]^{-1} - k_0^2[I])^{-1}$, $Z_0 = 1/Y_0$ is the free-space intrinsic impedance, and the superscript t denotes the transpose operation. As is shown in Appendix I, the coupling coefficients P_{ij} and Q_{ij} obey the orthogonality relations

$$(\beta_i^2 - \beta_j^{*2}) P_{ij} = 0 \quad (\beta_i^2 - \beta_j^2) Q_{ij} = 0. \quad (16)$$

All nondegenerate modes, both the complex and the noncomplex, are orthogonal in the Q sense. On the other hand, only noncomplex nondegenerate modes are orthogonal in the P sense. In particular, all complex modes have $P_{ii} = 0$, which means that an individual complex mode can carry neither active nor reactive power. Such a mode behaves in this aspect like modes at cutoff. Pairs of complex modes which have complex conjugate β^2 can just carry reactive power, as will be shown shortly. A similar result has been obtained for finline complex modes as a result of numerical investigations [17].

We will now investigate the modes corresponding to the two complex conjugate eigenvalues β_1^2 and $\beta_2^2 = (\beta_1^2)^*$. From the four possible modes, only two can be excited to the right of the plane of excitation. If we allow a very small but still finite amount of loss, it can be proved in a way similar to that in Appendix I that P_{ii} is very small but still finite. The propagation constants β_1 and β_2 of the modes, which can be excited to the right of the plane of excitation, must then show a negative imaginary part. (Note that the time and longitudinal dependence have been assumed to be $e^{+j\omega t}$ and $e^{-j\beta z}$, respectively.) The corresponding two roots of β_1^2 and β_2^2 can then be written as

$$\beta_1 = \beta' - j\alpha', \beta_2 = -\beta_1^* = -\beta' - j\alpha' \quad (17)$$

where α' and β' are positive. Due to the real nature of the characteristic matrix of (12), we have

$$A_2' = A_1'^* \quad B_2' = B_1'^*. \tag{18a}$$

Using (11), we get

$$A_2 = A_1^* \quad B_2 = -B_1^* \tag{18b}$$

and with (15), (17), and (18b), we obtain

$$P_{21} = -P_{12}^*. \tag{19}$$

This result has been used in [17] to show, in a general way, that a pair of complex modes can carry only pure reactive power so that they behave as a whole evanescently.

E. Backward-Wave Modes

Using (15) and (6a), the total power carried by a single mode can be written as

$$P = k_0\{(Z_0\beta^*)B^t B^* - (Y_0/\beta)D^t[C]^{-1}D^*\}. \tag{20}$$

Referring to (9b), let D_{cn} be the eigenvector of the nth TM cutoff mode, which corresponds to a cutoff wavenumber k_{cn}. Due to the real symmetric nature of $[S]$, the eigenvectors D_{cn}, $n=1,2,\cdots$, can be chosen to constitute a real orthonormal set [24]. The matrix $[W]$, whose nth column is D_{cn}, i.e.,

$$[W] = [D_{c1} \quad D_{c2} \cdots D_{cn} \cdots] \tag{21}$$

is then unitary. Because $[S]$ and $[C]$ are commutative, they share the same set of eigenvectors. It is then easily proved that D_{cn} is an eigenvector of $[C]^{-1}$, which corresponds to an eigenvalue $(k_{cn}^2 - k_0^2)$. Transforming D in (20) according to the unitary transformation

$$D = [W]\bar{D} \tag{22}$$

this equation can be written as

$$P = k_0\left\{(Z_0\beta^*)\sum_n |B_n|^2 + (Y_0/\beta)\sum_n (k_0^2 - k_{cn}^2)|\bar{D}_n|^2\right\} \tag{23}$$

where B_n and $\bar{D}_n$ are the nth elements of the column vectors B and $\bar{D}$, respectively.

Equation (23) shows that for propagating modes with real positive β, P is also real (as it should be) and can be either positive, which corresponds to forward-wave modes, or negative, which corresponds to backward-wave modes. In particular, let us investigate one of the modes, which becomes TM at cutoff, near its cutoff wavenumber (say k_{cm}). From (9), (21), and (22), it is easily shown that $B_n \to 0$, $\bar{D}_n \to K\delta_{nm}$ as $k_0 \to k_{cm}$, where K is a constant. If we assume that the term $(k_0^2 - k_{cm}^2)\cdot|K|^2$ dominates the other terms of (23) as $k_0 \to k_{cm}$, we get

$$P \to (k_0 Y_0/\beta)(k_0^2 - k_{cm}^2)|K|^2 \qquad \text{as } k_0 \to k_{cm}. \tag{24}$$

We have now two possibilities, which are shown in Fig. 2. In the first case, which is shown in Fig. 2(a), the mode is evanescent for $k_0 < k_{cm}$ and propagating for $k_0 > k_{cm}$. Equation (24) shows that the mode is capacitive for $k_0 < k_{cm}$, because $\beta = -j|\beta|$ and hence P is negative imaginary. For $k_0 > k_{cm}$, the mode represents a forward wave,

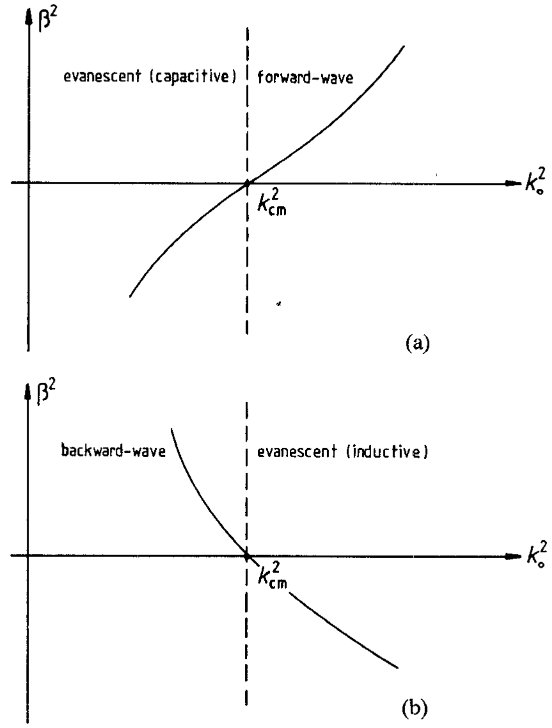

Fig. 2. The two possibilities for a mode which becomes TM at cutoff.

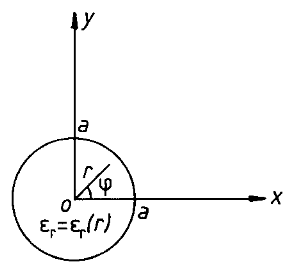

Fig. 3. The cross section of a rotational symmetrically filled circular waveguide.

because P and, hence, the group velocity are positive, which is confirmed by the increase of β versus k_0. In the second case, which is shown in Fig. 2(b), the mode is propagating for $k_0 < k_{cm}$ and evanescent for $k_0 > k_{cm}$. Equation (24) shows that the mode is inductive for $k_0 > k_{cm}$. For $k_0 < k_{cm}$, the mode represents a backward wave, because both P and the group velocity are negative. This is confirmed now by the decrease of β versus k_0.

F. Rotational Symmetrically Filled Circular Waveguide

This structure is the general form of the dielectric-rod-loaded circular waveguide, which most of the early publications about complex modes dealt with (e.g., [9]–[13]). The purpose of studying this special case is to explain some related features, e.g. why must complex modes show azimuthal dependence?

Consider the rotational symmetrically filled circular waveguide whose cross section is shown in Fig. 3. The relative permittivity of the filling medium is assumed to depend on the radial coordinate r only, i.e., $\epsilon_r = \epsilon_r(r)$. Each mode of the corresponding empty waveguide is characterized by two indices, one for the r dependence and the other for the φ dependence. The r dependence index will be used as a subscript, while that of the φ dependence is used as a superscript set in parentheses. The longitudinal electric and magnetic fields of the different empty-guide modes can then be written as

$$e_{zn}^{(i)} = V_n^{(i)} J_i\left(k_{ne}^{(i)} r\right) \sin\left(i\phi + \vartheta_n^{(i)}\right)$$
$$h_{zn}^{(i)} = I_n^{(i)} J_i\left(k_{nh}^{(i)} r\right) \sin\left(i\phi + \psi_n^{(i)}\right) \tag{25}$$

where J_i denotes the Bessel function of the first kind and ith order, and $V_n^{(i)}, I_n^{(i)}$ are normalizing constants. The column vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$ are written as

$$\boldsymbol{X} = \begin{bmatrix} \boldsymbol{X}^{(0)} \\ \boldsymbol{X}^{(1)} \\ \vdots \\ \boldsymbol{X}^{(i)} \\ \vdots \end{bmatrix} \tag{26}$$

where the elements of the subvector $\boldsymbol{X}^{(i)}$ are denoted by $X_n^{(i)}$. Matrices $[R^e]$, $[R^h]$, $[\Lambda^e]$, $[\Lambda^h]$, $[S]$, and $[T]$ are written as

$$[Z] = \begin{bmatrix} [Z^{(00)}] & [Z^{(01)}]\cdots[Z^{(0j)}]\cdots \\ [Z^{(10)}] & [Z^{(11)}]\cdots[Z^{(1j)}]\cdots \\ \vdots & \\ [Z^{(i0)}] & [Z^{(i1)}]\cdots[Z^{(ij)}]\cdots \\ \vdots & \end{bmatrix} \tag{27}$$

where the elements of the submatrix $[Z^{(ij)}]$ are denoted by $Z_{nm}^{(ij)}$.

Carrying out the integrations in (7) and keeping in mind that ϵ_r is φ independent, it is easily shown that all off-diagonal submatrices vanish. This confirms the fact that the radial inhomogeneity of the filling medium couples only those empty-guide modes which have the same azimuthal dependence. Equation (6) can then be written for the individual subvectors $\boldsymbol{A}^{(i)}$, $\boldsymbol{B}^{(i)}$, and $\boldsymbol{D}^{(i)}$, which represent the ith φ dependence, as

$$\left([I] - k_0^2[S^{(i)}]\right)\boldsymbol{D}^{(i)} = j\beta[S^{(i)}]\boldsymbol{A}^{(i)} \tag{28a}$$
$$[R^{e(i)}]\boldsymbol{A}^{(i)} - j\beta\boldsymbol{D}^{(i)} = -j\omega\mu_0[T^{(i)}]\boldsymbol{B}^{(i)} \tag{28b}$$
$$\left(k_0^2[R^{h(i)}] - [\Lambda^{h(i)}]\right)\boldsymbol{B}^{(i)} - \beta^2\boldsymbol{B}^{(i)} = j\omega\epsilon_0[T^{(i)}]^t\boldsymbol{A}^{(i)} \tag{28c}$$

where superscript (ii) has been replaced by just (i).

Modes without φ dependence (i.e., those for $i = 0$) have $[T^{(0)}] = 0$. This has the effect of decoupling the TE from the TM part of the field. Substituting $i = 0$ and $[T^{(0)}] = 0$ into (28) and eliminating $\boldsymbol{D}^{(0)}$, we get

$$\left(k_0^2[I] - [S^{(0)}]^{-1}\right)[R^{e(0)}]\boldsymbol{A}^{(0)} = \beta^2\boldsymbol{A}^{(0)} \tag{29a}$$
$$\left(k_0^2[R^{h(0)}] - [\Lambda^{h(0)}]\right)\boldsymbol{B}^{(0)} = \beta^2\boldsymbol{B}^{(0)}. \tag{29b}$$

Equation (29a) is the eigenvalue equation of the TM modes, whereas (29b) is that of the TE modes. Both have real symmetric characteristic matrices, which lead to real values of β^2 for both types of modes. We can conclude then that no complex modes can exist in the rotational symmetrically filled circular waveguide without showing azimuthal dependence.

G. Rectangular Waveguide with One-Dimensional Inhomogeneity

This structure represents a generalization of the single-slab or multislab-loaded rectangular waveguide, which is well investigated in the literature (see, e.g., [19], [21]–[23]). It is not, however, intended to produce (or reproduce) numerical results. The other methods, e.g., [21]–[23], are much more efficient in this aspect. Instead, we are aiming to confirm what has been stated in [19], namely that the modes of such a structure are all noncomplex.

Consider the rectangular waveguide whose cross section is shown in Fig. 4. The relative permittivity of the filling medium is assumed to be x dependent. Because each of the corresponding empty-guide modes is characterized by two indices, we use the x dependence index as a subscript and the y dependence index as a superscript set in parentheses. The longitudinal electric and magnetic field components of the empty-guide modes are given by

$$e_{zn}^{(i)} = V_n^{(i)} \sin(n\pi x/a)\sin(i\pi y/b)$$
$$h_{zn}^{(i)} = I_n^{(i)} \cos(n\pi x/a)\cos(i\pi y/b). \tag{30}$$

In the above equation, $i = 1,2,\cdots, n = 1,2,\cdots$ for $e_{zn}^{(i)}$ while $i = 0,1,2,\cdots, n = 0,1,2,\cdots$ for $h_{zn}^{(i)}$ ($i = n = 0$ is excluded).

Column vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$ can then be written as in (26). The subvectors of $\boldsymbol{A}$ and $\boldsymbol{D}$, which represent the TM part of the field, have $i = 1,2\cdots$; their elements have $n = 1,2,\cdots$. On the other hand, the subvectors of $\boldsymbol{B}$, which represent the TE part, have $i = 0,1,2,\cdots$ and their elements have $n = 0,1,2,\cdots$, except for the elements of $\boldsymbol{B}^{(0)}$, which have $n = 1,2,\cdots$.

Matrices $[R^e]$, $[R^h]$, $[\Lambda^e]$, $[\Lambda^h]$, $[S]$, and $[T]$ can also be written as in (27). The submatrices of $[R^e]$, $[\Lambda^e]$, and $[S]$ have $i = 1,2,\cdots, j = 1,2,\cdots$, and their elements have $n = 1,2,\cdots, m = 1,2,3,\cdots$. The submatrices of $[R^h]$ and $[\Lambda^h]$, on the other hand, have $i = 0,1,2,\cdots, j = 0,1,2,\cdots$, while their elements have $n = 0,1,2,\cdots, m = 0,1,2,\cdots$, with ($i = n = 0$) and ($j = m = 0$) being excluded. The submatrices of $[T]$, which represents the coupling between the TE and TM parts of the field, have $i = 1,2,\cdots, j = 0,1,2,\cdots$, and their elements have $n = 1,2,\cdots, m = 0,1,2,\cdots$, with ($j = m = 0$) being excluded.

Performing the integrations of (7) and taking into account that ϵ_r does not depend on y, it is readily proved that all off-diagonal submatrices or, more precisely, the submatrices having $i \neq j$, vanish. Empty-guide modes with different y dependence are then decoupled. Equation (6) can then be separately written for the individual components showing the same y dependence. For $i = 0$, (6) is reduced to

$$\left(k_0^2[R^{h(0)}] - [\Lambda^{h(0)}]\right)\boldsymbol{B}^{(0)} = \beta^2\boldsymbol{B}^{(0)} \tag{31}$$

where superscript (00) has been replaced by just (0). This confirms that modes which are y independent must show a

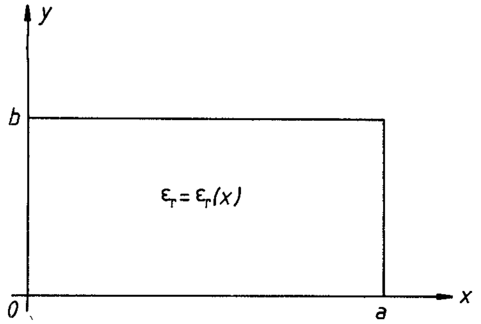

Fig. 4. The cross section of a rectangular waveguide with x-dependent ϵ_r.

pure TE field. The characteristic matrix of (31) is real and symmetric, and all its eigenvalues β^2 are then real.

For $i \neq 0$, (6) is reduced to an equation which is similar to (28) with superscript (ii) being replaced by (i). To simplify the notation, superscript (i) will be omitted from now on. Equation (6) will then be used in the sense that all matrices and column vectors denote the ith submatrices and subvectors, respectively.

The different structures of the matrices and column vectors representing TE and TM parts leads to some difficulties, which can be overcome if we carry out the following modifications. Column vectors $\boldsymbol{A}$ and $\boldsymbol{D}$ are modified to $\tilde{\boldsymbol{A}}$ and $\tilde{\boldsymbol{D}}$, respectively, according to

$$\tilde{\boldsymbol{X}} = \begin{bmatrix} 0 \\ \boldsymbol{X} \end{bmatrix}. \tag{32a}$$

Let $\boldsymbol{N}$ denote a column vector with zero elements. Matrices $[R^e]$, $[\Lambda^e]$, and $[S]$ are then modified to $[\tilde{R}^e]$, $[\tilde{\Lambda}^e]$, and $[\tilde{S}]$, respectively, according to

$$[\tilde{Z}] = \begin{bmatrix} 0 & \boldsymbol{N}^t \\ \boldsymbol{N} & [Z] \end{bmatrix}. \tag{32b}$$

The coupling matrix $[T]$ is finally modified to $[\hat{T}]$ by

$$[\hat{T}] = \begin{bmatrix} \tilde{\boldsymbol{N}}^t \\ [T] \end{bmatrix}. \tag{32c}$$

Matrices such as $[\tilde{Z}]$ are in fact singular. They can, however, be inverted if we use their special algebra, which is presented in Appendix II. Equation (6) can now be written with the modified matrices and vectors as

$$\left([\tilde{I}] - k_0^2[\tilde{S}]\right)\tilde{\boldsymbol{D}} = j\beta[\tilde{S}]\tilde{\boldsymbol{A}} \tag{33a}$$

$$[\tilde{R}^e]\tilde{\boldsymbol{A}} - j\beta\tilde{\boldsymbol{D}} = -j\omega\mu_0[\hat{T}]\boldsymbol{B} \tag{33b}$$

$$\left(k_0^2[R^h] - [\Lambda^h]\right)\boldsymbol{B} - \beta^2\boldsymbol{B} = j\omega\epsilon_0[\hat{T}]^t\tilde{\boldsymbol{A}}. \tag{33c}$$

Adjusting the normalization coefficients V_n and I_n of (30) according to (3), all matrices of (33) can be written in terms of only three matrices, namely $[\Lambda^n]$, $[\tilde{F}^s]$, and $[F^c]$. $[\Lambda^n]$ is a diagonal matrix with elements $(n\pi/a), n = 0,1,2,\cdots$. The elements of $[F^s]$ and $[F^c]$ are given by

$$F_{nm}^s = (2/ak_nk_m)\int_0^a \epsilon_r(x)\sin(n\pi x/a)\sin(m\pi x/a)\,dx$$

$$F_{nm}^c = \left(\sqrt{\epsilon_{0n}\epsilon_{0m}}/ak_nk_m\right)\int_0^a \epsilon_r(x)\cos(n\pi x/a) \cdot\cos(m\pi x/a)\,dx \tag{34}$$

where $\epsilon_{0n} = (2-\delta_{0n})$ and $k_n^2 = k_{ne}^2 = k_{nh}^2 = (i\pi/b)^2 + (n\pi/a)^2$. The matrices of (33) are related to $[\Lambda^n]$, $[\tilde{F}^s]$, and $[F^c]$ by

$$\begin{aligned}
[\Lambda^h] &= (i\pi/b)^2[I] + [\Lambda^n]^2 \\
[\tilde{\Lambda}^e] &= (i\pi/b)^2[\tilde{I}] + [\Lambda^n]^2 \\
[R^h] &= [\Lambda^n][\tilde{F}^s][\Lambda^n] + (i\pi/b)^2[F^c] \\
[\tilde{R}^e] &= [\Lambda^n][F^c][\Lambda^n] + (i\pi/b)^2[\tilde{F}^s] \\
[\tilde{S}] &= [\tilde{F}^s] \\
[\hat{T}] &= (i\pi/b)\left([\tilde{F}^s][\Lambda^n] - [\Lambda^n][F^c]\right).
\end{aligned} \tag{35}$$

Now let $\boldsymbol{V}^{(x)}$ and $\boldsymbol{I}^{(x)}$ be two column vectors whose elements are proportional to the series expansion coefficients of the electric and magnetic field x components E_x and H_x, respectively. Using (4), these vectors are related to $\tilde{\boldsymbol{A}}$, $\boldsymbol{B}$, and $\tilde{\boldsymbol{D}}$ by

$$\begin{aligned}
\boldsymbol{V}^{(x)} &= j[\Lambda^n]\tilde{\boldsymbol{A}} + \omega\mu_0(i\pi/b)\boldsymbol{B} \\
\tilde{\boldsymbol{I}}^{(x)} &= \beta[\Lambda^n]\boldsymbol{B} + \omega\epsilon_0(i\pi/b)\tilde{\boldsymbol{D}}.
\end{aligned} \tag{36}$$

Substituting (35) into (33) and using (36), the following two decoupled equations are obtained:

$$[\Lambda^e]\left(k_0^2[S] - [I]\right)\boldsymbol{I}^{(x)} = \beta^2\boldsymbol{I}^{(x)} \tag{37a}$$

$$\left(k_0^2[\Lambda^h] - [\Lambda^n][\tilde{F}^s]^{-1}[\Lambda^n]\right)[F^c]\boldsymbol{V}^{(x)} = \left(\beta^2 + (i\pi/b)^2\right)\boldsymbol{V}^{(x)}. \tag{37b}$$

Equations (37a) and (37b) are, in fact, the eigenvalue equations of the LSE and LSM modes, respectively [19]. Both have real symmetric characteristic matrices, which means that all eigenvalues are real. Equations (31) and (37) confirm what has been formulated in [19] by using a completely different approach, namely that the modes of a rectangular waveguide with one-dimensional inhomogeneity are of either the LSE or the LSM type and that they are always noncomplex modes; i.e., they always have real values of β^2.

III. Homogeneously Filled Waveguides with Anisotropic Media

Consider now the waveguide shown in Fig. 1 being homogeneously filled with a medium with permittivity tensor

$$[\epsilon_r] = \begin{bmatrix} \epsilon_t & +j\kappa & 0 \\ -j\kappa & \epsilon_t & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix}. \tag{38}$$

The relative permeability is still assumed to be unity. Equation (1) then reads

$$\boldsymbol{J} = j\omega\epsilon_0\left([\epsilon_r] - [I]\right)\boldsymbol{E} \tag{39}$$

where $[I]$ is the idem factor (unit dyadic). The longitudinal and transverse components of the polarization current $\boldsymbol{J}$ can then be written as

$$\begin{aligned}
J_z &= j\omega\epsilon_0(\epsilon_z - 1)E_z \\
\boldsymbol{J}_t &= j\omega\epsilon_0\{(\epsilon_t - 1)\boldsymbol{E}_t + j\kappa(\boldsymbol{E}_t \times \hat{\boldsymbol{k}})\}.
\end{aligned} \tag{40}$$

Substituting (4) and (40) into Maxwell's equations and making use of (2) and (3), the following system of matrix equations is obtained, which relates the column vectors $\boldsymbol{A}$, $\boldsymbol{B}$, and $\boldsymbol{D}$:

$$j\beta\epsilon_z \boldsymbol{A} = ([\Lambda^e] - \epsilon_z k_0^2[I])\boldsymbol{D} \tag{41a}$$

$$\epsilon_t \boldsymbol{A} - j\beta \boldsymbol{D} = -\omega\mu_0\kappa[U]\boldsymbol{B} \tag{41b}$$

$$[U]\{[\Lambda^h] + (\beta^2 - \epsilon_t k_0^2)[I]\}\boldsymbol{B} = \omega\epsilon_0\kappa \boldsymbol{A}. \tag{41c}$$

Here, $[U]$ is a real nonsymmetric matrix whose elements are given by

$$U_{nm} = (1/k_{ne}k_{mh})\int_S (\nabla_t e_{zn} \cdot \nabla_t h_{zm})\, dS. \tag{42}$$

Due to the individual completeness property of the infinite sets $\{e_{zn}\}$ and $\{h_{zm}\}$, each e_{zn} can be expanded over the set $\{h_{zm}\}$ and vice versa. The series expansion coefficients for both cases are proportional to U_{nm}. Carrying out the gradient operation on these expansions and taking care of the step discontinuities which may exist at the contour enclosing the guide cross section, the following identity can be proved:

$$[U][\Lambda^h] = [\Lambda^e][U]. \tag{43}$$

Substituting (43) into (41), the following eigenvalue equation is obtained:

$$\begin{bmatrix} (\epsilon_t/\epsilon_z)(\epsilon_z k_0^2[I] - [\Lambda^e]) & \omega\mu_0(\kappa/\epsilon_z)(\epsilon_z k_0^2[I] - [\Lambda^e]) \\ \omega\epsilon_0\kappa[I] & (\epsilon_t k_0^2[I] - [\Lambda^e]) \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{A} \\ [U]\boldsymbol{B} \end{bmatrix} = \beta^2 \begin{bmatrix} \boldsymbol{A} \\ [U]\boldsymbol{B} \end{bmatrix}. \tag{44}$$

Its characteristic matrix is again real and nonsymmetric. Complex modes are consequently possible in anisotropically filled waveguides.

Most of the results which have been obtained for inhomogeneously filled waveguides are equally valid for anisotropically filled waveguides. Modes at cutoff and backward-wave modes in both structures have similar characteristics. Hence, no further investigations are needed for anisotropically filled waveguides.

IV. Conclusions

Inhomogeneously filled and anisotropically filled lossless waveguides of arbitrarily shaped cross section have been rigorously analyzed. It has been shown that complex and backward-wave modes can be supported by these structures. Modes at cutoff have been shown to have real cutoff frequencies and to be either of TE or of TM type. different orthogonality relations have been investigated. The modes of rotational symmetrically filled circular waveguides which show no azimuthal dependence have been shown to be noncomplex and either TE or TM. Modes of rectangular waveguides with one-dimensional inhomogeneity are either of LSE or of LSM type. No complex modes can be supported by these structures.

Appendix I

To prove the orthogonality relations (16), we rewrite (6) in the following form:

$$\boldsymbol{D}_i = j\beta_i[C]\boldsymbol{A}_i \tag{A1}$$

$$[R^e]\boldsymbol{A}_i + \beta_i^2[C]\boldsymbol{A}_i = -jk_0Z_0[T]\boldsymbol{B}_i \tag{A2}$$

$$([\Lambda^h] - k_0^2[R^h])\boldsymbol{B}_i + \beta_i^2\boldsymbol{B}_i = -jk_0Y_0[T]^t\boldsymbol{A}_i \tag{A3}$$

where subscript i denotes the ith mode. Conjugating (A2) for the jth mode and premultiplying by $Y_0\boldsymbol{A}_i^t$, we get

$$Y_0\boldsymbol{A}_i^t[R^e]\boldsymbol{A}_j^* + Y_0\beta_j^{*2}\boldsymbol{A}_i^t[C]\boldsymbol{A}_j^* = jk_0\boldsymbol{A}_i^t[T]\boldsymbol{B}_j^*. \tag{A4}$$

Transposing (A3) for the ith mode and postmultiplying by $Z_0\boldsymbol{B}_j^*$ results in

$$Z_0\boldsymbol{B}_i^t([\Lambda^h] - k_0^2[R^h])\boldsymbol{B}_j^* + Z_0\beta_i^2\boldsymbol{B}_i^t\boldsymbol{B}_j^* = -jk_0\boldsymbol{A}_i^t[T]\boldsymbol{B}_j^*. \tag{A5}$$

By adding (A4) and (A5), we find

$$Z_0\beta_i^2\boldsymbol{B}_i^t\boldsymbol{B}_j^* + Y_0\beta_j^{*2}\boldsymbol{A}_i^t[C]\boldsymbol{A}_j^* = -Z_0\boldsymbol{B}_i^t([\Lambda^h] - k_0^2[R^h])\boldsymbol{B}_j^* - Y_0\boldsymbol{A}_i^t[R^e]\boldsymbol{A}_j^*. \tag{A6}$$

Subscripts i and j in (A6) can be interchanged. Taking then the transposed conjugate form results in

$$Z_0\beta_j^{*2}\boldsymbol{B}_i^t\boldsymbol{B}_j^* + Y_0\beta_i^2\boldsymbol{A}_i^t[C]\boldsymbol{A}_j^* = -Z_0\boldsymbol{B}_i^t([\Lambda^h] - k_0^2[R^h])\boldsymbol{B}_j^* - Y_0\boldsymbol{A}_i^t[R^e]\boldsymbol{A}_j^*. \tag{A7}$$

From a comparison between (A6) and (A7), we find now

$$(\beta_i^2 - \beta_j^{*2})(Z_0\boldsymbol{B}_i^t\boldsymbol{B}_j^* - Y_0\boldsymbol{A}_i^t[C]\boldsymbol{A}_j^*) = 0. \tag{A8}$$

The procedure described so far is repeated but without taking the complex conjugate forms. This yields

$$(\beta_i^2 - \beta_j^2)(Z_0\boldsymbol{B}_i^t\boldsymbol{B}_j + Y_0\boldsymbol{A}_i^t[C]\boldsymbol{A}_j) = 0. \tag{A9}$$

The orthogonality relations (16) now follow directly from a comparison between (A8), (A9), and (15).

Appendix II

Let $[K]$ and $[L]$ be $(N \times N)$ square matrices and $[M]$ be an $(N \times (N+1))$ matrix. Let also $\boldsymbol{X}$ and $\boldsymbol{Y}$ be an $(N \times 1)$ and an $((N+1) \times 1)$ column vector, respectively, and let $\boldsymbol{N}$ be an $(N \times 1)$ column vector whose elements are all zero. Using (32), $\tilde{\boldsymbol{X}}$, $\boldsymbol{Y}$, and $\tilde{\boldsymbol{N}}$ are all $((N+1) \times 1)$ column vectors, while $[\tilde{K}]$, $[\tilde{L}]$, and $[\hat{M}]$ are all $((N+1) \times (N+1))$ square matrices. We will now prove some identities:

$$[\tilde{K}] \cdot [\tilde{L}] = \begin{bmatrix} 0 & \boldsymbol{N}^t \\ \boldsymbol{N} & [K] \end{bmatrix} \cdot \begin{bmatrix} 0 & \boldsymbol{N}^t \\ \boldsymbol{N} & [L] \end{bmatrix} = \begin{bmatrix} 0 & \boldsymbol{N}^t \\ \boldsymbol{N} & [K]\cdot[L] \end{bmatrix} = (\widetilde{[K]\cdot[L]}). \tag{A10}$$

In particular, if $[L] = [K]^{-1}$, $[\tilde{L}]$ plays the same role as the inverse of $[\tilde{K}]$ with respect to the modified identity matrix $[\tilde{I}]$:

$$[\tilde{K}]\cdot\tilde{X}=\begin{bmatrix}0 & N^t\\ N & [K]\end{bmatrix}\cdot\begin{bmatrix}0\\ X\end{bmatrix}=\begin{bmatrix}0\\ [K]\cdot X\end{bmatrix} = \left(\widetilde{[K]\cdot X}\right) \quad \text{(A11)}$$

$$[\hat{M}]\cdot Y=\begin{bmatrix}\tilde{N}^t\\ [M]\end{bmatrix}\cdot Y=\begin{bmatrix}0\\ [M]\cdot Y\end{bmatrix} = \left(\widetilde{[M]\cdot Y}\right) \quad \text{(A12)}$$

$$[\hat{M}]^t\cdot\tilde{X}=\begin{bmatrix}\tilde{N} & [M]^t\end{bmatrix}\cdot\begin{bmatrix}0\\ X\end{bmatrix} = [M]^t\cdot X. \quad \text{(A13)}$$

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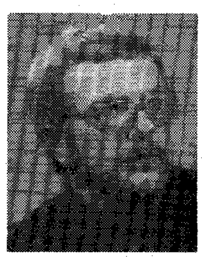

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